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REDUCTION OF NOISE AND BIAS IN RANDOMLY SAMPLED POWER SPECTRA

^{#1}PRAVEEN KUMAR VOLADRI, Research Scholar, ^{#2}Dr. JAGADEESH BODAPATI, Supervisor, ^{#3}Dr. ARRAMARAJU PRASAD RAJU, Co-Supervisor, Department of Electronics and Communication Engineering, NIILM UNIVERSITY, KAITHAL, HARYANA, INDIA.

ABSTRACT: We investigate the sources of anomalies and noise in power spectrum projections based on randomly selected data. In specifically, speed data from a burst-mode laser Doppler anemometer is examined. The work suggests new approaches to dealing with issues like as noise and spectral bias. These include solving challenges that arise when the optimum Poisson sample rate varies due to dead time effects and sample rate-velocity correlations. Researchers have already discovered that the effects of noise and dead time in recordings with a fixed length of time are comparable to those detected in recordings with an infinite length of time and group averages. We demonstrate that the measured sampling function may be employed in a deconvolution method to remove noise and dead time from the spectra of finite recordings. We also provide a novel power spectrum predictor that employs a sped-up version of the slotted auto-covariance algorithm.

Keywords: Noise And Distortions, Power Spectral, Randomly Sampled Data, Randomly Sampled Data

1. INTRODUCTION

Several academic studies have used arbitrary data to determine the power spectrum of a dynamic process. The previously cited publications include those by Albrecht et al. (2003), R. B. Blackman and J. W. Tukey (1958), H.S. Shapiro and R.A. Silverman (1960), and Durst et al.

Each has distinct advantages and disadvantages in general.

It is possible to acquire power spectrum values that are alias-free, sufficiently stochastic (for example, a Poisson process), and independent of the process being sampled. Estimates derived from similarly selected often occurring data have lower variability. This is partly due to the sample technique's random nature, which demands additional data or a longer sampling time. Several power spectral estimators have tackled the issue of laser Doppler measurements of turbulence spectra. Numerous studies have been conducted to assess the advantages and disadvantages of various tactics, but no final result has been reached.

Gaster and Roberts devised the direct technique for measuring the power spectrum (PS) using a modified periodogram in the early 1970s (1977). In 1978, Masry undertook more study to determine the significance of aliasing-free bands. Researchers Roberts and Gaster (1980) and Roberts et al. (1980) investigated the methodologies used in LDA processors. Buchhave et al. created the first weighted PS algorithms with residence time in 1979. These algorithms destroyed the link between sample rate and performance. Gaster and Roberts (1975) offered an alternative strategy based on Mayo's work (1974). To obtain the PS, they first computed the autocovariance and then used a Fourier transform. There are several issues that early estimators face when dealing with big variances at high frequencies. Several studies (e.g., Tropea, 1995; Benedict, Nobach, and Tropea, 2000) eventually addressed this issue. The importance of fuzzy slot width must be highlighted. Several writers have updated the slotted autocovariance approach (SACF), including Tummers and Passchier (1996), van Maanen et al. (1999), and Nobach (1998). One of the planned changes is to the slot width. Nobach (2002) and Benedict et al. (2000) discuss slotted correlation function real-time estimates. Benak et al. (1993) explored the link between particle rate and slot breadth.

Another popular strategy is the "sample-and-hold method," which interpolates velocity data. Adrian (1987) shown in a previous work that resampling a detected velocity value can dramatically affect the computed spectrum, resulting in a different measurement. Following that, many interpolation approaches and filter function correction reconstruction efforts were investigated, including those of Simon (2004) and Moreau (2011).

Many advances have been achieved in estimating the power spectrum from LDA data using these techniques; however, for the sake of this study, we shall confine our use to the basic estimators of the straight technique and the SACF method. To correct spectral bias caused by the sample rate in the spectrum, the use of a raw sampling device becomes critical. Refining and filtering techniques are likely to modify the sample function, making it useless for rectification approaches. In fact, the raw data clarity of the signal reduces the accuracy of spectrum computations. It is more likely that problems with data processing, rather than the power spectrum in general, will be discovered during future efforts to improve quality.

Overall, the theory is presented in a very simple fashion; the samples are envisioned as point processes (particularly, delta function samples), and the conclusions are achieved through ensemble averages across an infinite number of records or recordings of infinite length. We want to begin with a limited collection of previously measured places, just like with real measurements. Examining potential real-world concerns that could have a major impact on spectral estimates is also critical. Examples include the effects of instrumental dead time, random sampling noise, and finite width sample pulses.

The primary goal of this research project is to study the feasibility of employing deconvolution techniques to rectify the power spectrum produced from observational data. The final power spectrum is generally formed by combining a number of spectral filters established by noise introduction or sampling rate changes. To adjust the spectrum, we analyze numerous effects in time delay space and divide the measured function's autocovariance by the autocovariance of the sample technique. Deconvolution is obviously tough, especially when dealing with noisy data, and diverse approaches may not always be beneficial. It has been discovered possible to reduce some of the bias and noise caused by less-than-ideal measurement procedures.

The impact of biases and noise on power spectra from real instruments will be discussed in further detail in the following sections. In addition, we will use tactics such as deconvolution to counteract these consequences. Our experiments into turbulent velocity power spectra using the laser Doppler anemometer (LDA) yielded interesting results. Furthermore, we are confident that our approaches will be beneficial in solving a wide range of difficulties. Some of the statistics came from recent studies on dead time effects in power spectrum estimation (Frontera and Fuligni, 1978; Zhang et al., 1995; Buchhave et al., 2014; and Velte et al., 2014b). In contrast, this study does practical evaluations using a small sample size utilizing burst-mode LDA. To accomplish this, we offer a unique notion called the noise function, which evaluates the unpredictability of a particular collection of data.

2. BACKGROUND WORK

Properties of LDA data

This study will focus on the features of LDA data that affect power spectrum estimation. We are particularly interested in burst-mode LDA, which generates a single velocity data point when a seed particle passes through the measuring volume formed by the intersection of two coherent laser beams. It is also believed that the processor calculates the particle's arrival time and motion duration within the measuring space. The latter is known as "residence time." It is assumed that the particles are evenly distributed throughout the fluid. With the appropriate volume reduction and consistent speed, there is little likelihood of detecting many particles at the same time. This procedure is called Poisson sampling. Despite this, the velocity bias causes the sample rate to vary in lockstep with the velocity. This occurs when particles move faster across the measurement space than they would at a slower rate.

Starting here, the sampling rate is assumed to be proportional to the magnitude of the instantaneous motion within the volume in question. It is also believed that residence time weighted (RTW) techniques are used to compute mean velocity, velocity autocovariance function (ACF), and velocity power spectrum (PS). Using these strategies, velocity bias can be reduced from statistical results (George et al., 1978; Buchhave et al., 1979; Velte et al., 2014a). Consider speed as a new, objective speed data point that may be used in statistical computations to acquire a better understanding of the concept. To do this, multiply the velocity by the time of the stay. In the subsequent theoretical analysis, this number will replace the measured velocity data point. As a result, the process under study and the sampling approach might be regarded statistically separate. Figure 1 depicts the theories behind the averaging and inactive time effects.



Figure 1. The sampling process (Buchhave (2014).

The image shows the envelope of a Doppler burst, which is an electrical signal generated when a particle enters the measurement chamber and is detected by the receiver as Doppler-modulated light. Using the augmented and filtered detector signals, the signal processor derives the modulated discharge's Doppler frequency. When the signal hits the burst detection threshold, the signal processor begins rapidly digitizing it. An FFT analysis will then be performed to generate a calibrated velocity data point. The collected velocity data point is then transferred to the data processor or an intermediate buffer storage for further processing. Given that computing the Doppler frequency needs a certain number of digital samples, the image shows that the processing time (tp) is constant.

We'll assume that the task's average pace during execution reflects the impact of this processing time. Buchhave (2014) discovered that the high frequency section of the spectrum is filtered, and the average decreases. The residence time ts, also known as the burst length, is related to the maximum processing time that can occur so that tp is less than ts. To account for the minimal ts predicted by the measurement, the actual processing time must be adjusted. The signal processor is unable to evaluate the signal of the next particle because it falls below the threshold until the burst detector is restarted. This is referred to as "dead time." The figure shows that the signal processor stops operating during the inactive time, or td. Because of this, the dead time effect may affect the power spectrum estimate, and the latency used to generate the ACF estimate must be shorter than the delay between samples. (2014) The author is Buchhave. Accurate LDA measurements of turbulence are difficult to achieve due to light interference between two or more particles in the measuring volume, as well as significant particle speed variation, which both impact residence time measurements. As indicated in Velte (2014b), more research into the influence of inactive time complicates the model and computations. The idle period causes an unwanted loss in power at the low end as well as an oscillation at the high end of the spectrum. Following that, we will look at the features of random sampling noise and how noise, filtering, and dead time interact to generate the spectrum.

3. RANDOM SAMPLING NOISE

Frequency content of the noise

Reexamine Equation (1) to determine the consequences of scattering the samples.

$$\hat{S}_{0}(f) = S_{u'}(f) \otimes \frac{1}{\nu \bar{N}} \sum_{k,k'}^{N} e^{-i2\pi f \tau_{kk'}} = \frac{\overline{u'^{2}}}{\nu} + S_{u'}(f) \otimes \frac{1}{\nu \bar{N}} \sum_{k\neq k'}^{N} e^{-i2\pi f \tau_{kk'}}$$
(1)

The second component, a convolution, consists of the real spectrum and a zero-mean noise term made up of sampling exponentials with random phases and distinct, predictable fluctuations. The spectral estimate corresponds to the measurement's $\tau_{kk'}$ noise levels. Convolution, which is equivalent to the restricted resolution that may be achieved with a limited set of routinely sampled data, broadens the spectrum when the record length is limited.

The convolution with a potential mean velocity is eventually found to be added to the noise term.

$$S_{u}(f) \otimes \frac{1}{\nu \overline{N}} \sum_{k \neq k'}^{N} e^{-i2\pi f \tau_{kk'}} = \left[\overline{u}^{2} \delta(f) + S_{u'}(f)\right] \otimes \frac{1}{\nu N} \sum_{k \neq k'}^{N} e^{-i2\pi f \tau_{kk'}}$$
(2)

In light of this, the power spectral estimate should be used after subtracting the mean. This inquiry produces all spectrum values for the portion of velocity that varies.



Figure 2 shows the spectral distribution, which includes the determined offset (broken line), frequency-dependent noise (cross-hatched area), and the overall observed spectrum (solid line).

Examine the noise function or the convolution integral of the cross terms to determine this

$$S_{u'}(f) \otimes \frac{1}{\nu \bar{N}} \sum_{k \neq k'}^{N} e^{-i2\pi f \tau_{u'}} = \int_{-\infty}^{\infty} S_{u'}(f') \frac{1}{\nu \bar{N}} \sum_{k \neq k'}^{N} e^{-i2\pi (f-f')\tau_{u'}} df',$$
(3)

The frequency spectrum of the noise shows the dispersion of the velocity spectrum; noise becomes more noticeable as the spectrum expands. The cross-hatched region in Figure 2 shows how the power spectrum estimate was distorted by the small sample size. The combination of the spectrum and noise function generates high frequency noise in the form of a narrow spectral band. Low frequency noise, on the other hand, is caused by wide-band operation. The sinc-squared frequency function eventually blends with the spectrum as the sample rate rises and the record length remains constant. As a result, the commotion decreases. Because of the horizontal temporal window, this happens. Figure 3 depicts the power spectrum of a spectral line with a single record calculated using the direct technique at both a low sample rate (in the middle) and a high sample rate (on the left). When the real spectrum and the noise function are merged, the range of frequencies in the noise changes, indicating that the true spectrum has a limited frequency range. The introduction of self-

products results in a spectrum shift in all three cases. The variance equation shows that the high frequency section of the spectrum, which is unaffected by the true spectrum, has two components: a constant spectral offset and a mean fluctuation of zero caused by the stochastic nature of the sample arrival times. It is clear that the spectrum keeps a value greater than zero even after the self-products are deleted. This is in contrast to a spectrum in which the true spectrum approaches zero at frequencies while the spectrum excluding the self-products fluctuates around zero due to random sampling noise.



Figure 3 depicts the power spectrum of a minute amount of computer-generated velocity data. A thin spectral line and a high sampling rate are shown on the left. The midway is defined by a thin spectral line and a low sampling rate. The sample rate is low, and the line to the right is broad.

4. CORRECTING THE POWER SPECTRUM BY DECONVOLUTION

Theory

A variety of factors have been shown to skew the observed power range. According to multiple sources, the real spectrum is multiplied by a sinc-squared transfer function to account for the processing time (tp) required to digitize and analyze the Doppler burst while measuring velocity. If you understand TP, you should be able to get an accurate spectrum estimate by simply dividing the frequency space by the transfer function.:

$$\hat{S}_0(f) = \hat{S}_{0,\Delta t_p}(f) / \operatorname{sinc}^2(\pi f \Delta t_p)$$
(4)

However, its overall influence is far lower than that of inactive time.

Regaining lost time. A potential dead time is represented in the frequency space of the real

spectrum by combining a convolution and a dead time function, assuming the averaging effect is taken into account. To apply the necessary correction, divide the correlation space by a known ACF dead time. A predetermined inactive time is required for this strategy to be much more user-friendly (Buchhave, 2014). Nonetheless, the variation in dwell time in empirical LDA data complicates this process (Velte, 2014b). However, by using the sampling function ACF, you can change the PS for any method that affects the sample rate, even those affected by dead time and noise.

Correcting for random sampling noise

As illustrated in the given source, random sampling noise can be thought of as a mixture of the record-specific noise function and the expected real spectrum after accounting for dead time and filtering. Noise is created when a velocity ACF is multiplied by a noise ACF in correlation space. To eliminate noise during the reconstruction process, the recorded velocity ACF would be divided by the noise function ACF.

$$\hat{C}_{0,correced}(\tau) = \hat{C}_0(\tau) / \hat{C}_g(\tau).$$
(5)

The good news is that the arrival dates from the measured record can be used for deconvolution.

Deconvolution by means of the measured sampling function ACF

The ACF, or measured sampling function, has the ability to minimize the detrimental impacts outlined above. When considering the filtered spectrum $,S_0\Delta t_p$, t is possible to mix up the dead time and noise functions.

$$S_{0,\Delta t_{p}}(f) \otimes \left[\delta(f) - 2\Delta t_{d}\operatorname{sinc}(2\pi f\Delta t_{d})\right] \otimes \frac{1}{\nu \overline{N}} \sum_{k \neq k'}^{N} e^{-i2\pi f \tau_{kk'}}$$
(6)

Multiplying three ACFs in correlation space yields the following results:

$$\hat{C}_{0,\mathcal{M}_{p}}(\tau) \cdot \left[1 - \Pi_{\mathcal{M}_{d}}(\tau)\right] \cdot \hat{C}_{g}(\tau) \equiv \hat{C}_{0,\mathcal{M}_{p}}(\tau) \cdot \hat{C}_{g}^{M}(\tau)$$
(7)

ACF stands $\hat{C}_{g}^{M}(\tau)$ for the measured sampling function.. To get a spectrum free of both dead time and noise, divide the spectrum in correlation space by the observed sampling function ACF.

$$\hat{C}_{u}(\tau) = \hat{C}_{0}(\tau) / \hat{C}_{g}^{M}(\tau) .$$
(8)

Indeed, the ACF (measured sampling function) includes all variables that can influence the sample rate, such as dead time effects, electronic filtering effects, and random sampling. The lack of velocity bias can be due to dwell time weighting. Given that the measured sampling

function is the product of two delta functions, it is unclear how this division can be carried out. The following two sections will go over two different techniques to carrying out the plan. It was expected that different spectral estimators would produce diverse dead time and noise effects.

Implementation of Deconvolution

Real-world programming use a variety of methods for measuring the power spectrum. Following that, the slotted autocovariance method and direct deconvolution techniques will be discussed. To validate the results, the methodologies will be applied to velocity data collected from a turbulent free jet in the air, as well as computer-generated velocity data. Buchhave delivers in-depth analysis of computer-generated data. A Poisson process and a Von Karman power spectrum were used to create random samples from a high-speed main velocity record. The Von Karman spectrum was chosen to simulate the jet spectrum with extraordinary accuracy.

$$S_{\nu \kappa}(f) = \frac{1}{62.5} \cdot \frac{1}{\left(1 + \left(f/45\right)^2\right)^{5/6}} \cdot \exp\left(-\left(f/2500\right)^{4/3}\right).$$
(9)

Using the formula $l = 2 \operatorname{var}(u) / S(0)$, one can calculate the integral time scale of this process. In response to performance, we adjusted the Poisson process to produce only ones and zeros (with very few twos). This was done so that velocity bias may be incorporated. Despite the presence of additional phase noise on the computer systems, this data was excluded from the presentation. The insertion of it would simply raise the spectrum's noise threshold. Velte (2014a) provided an overview of the jet data.

Directly using data generated by computers Prior to selecting speed samples at random, the power spectrum is computed utilizing a DFT with frequencies that are equally spaced according to a random sampling function. Following that, significant ACFs are found by performing an inverted FFT on these spectra. Following that, we divide in accordance with Equation (8) before returning to frequency space via FFT. Following the repeat of the method for M recordings, the final spectrum is calculated using block averaging. The sample function ACF could have zero or negative values due to random sampling noise, which could be troublesome for this approach. Such values may cause instability into the division process. Individuals typically apply a Wiener deconvolution, which includes appending the mean square noise to the denominator, or a small, constant value to C_g n order to sidestep this difficulty.(As it happens, the constant is unnecessary when the block average contains a large number of records. The process of averaging eliminates the sharpness associated with

infinities and negative spectral values. The power spectrum of the computer-generated data collection depicted in Figure 4 can be obtained through the utilization of the following parameters: a mean speed of 5 ms-1, a turbulence strength of 25%, a record length of 1 s, an average of 200 recordings per block, a sampling rate of 4500 s-1, and a measuring volume diameter of 40 m.



The power range of the computer-generated velocity data is illustrated in Figure 4. The power spectrum subsequent to deconvolution is depicted in red, while the Von Karman model spectrum is represented in green. The direct procedure is visually represented in blue. Both have, on average, over two hundred records. Three shapes are depicted in the image: The yellow trace represents the Von Karman spectrum model, upon which the computer-generated data was derived. The direct method was utilized to compute the power spectrum, which is visually represented by the blue graph. In contrast to the blue curve, the red curve represents the result of the deconvolution procedure that was previously discussed. It is evident that careful consideration has been given to the impact of inactive time. Although deconvolution may appear to introduce additional noise, it is critical to bear in mind that the observed image is logarithmic in nature, indicating that alterations are more conspicuous at lower frequencies. A block average of two hundred records eliminates additional disturbance. Both the red and blue spectra are generated using identical source data in an identical manner.

5. CONCLUSION

A number of effects that would occur in an actual experiment could be described by describing data extracted at random from a burst-mode LDA using a logical sampling function. These encompass dead time effects, which occur when the processor is unable to generate data with an excessively short interval between samples, averaging due to

the constrained processing time required to calculate velocity, and the noise function, which provides details regarding noise in time delay space or frequency space for a given record. Furthermore, we provided evidence that the intended power spectrum estimation of a solitary velocity record can be calculated by integrating the real power spectrum, a "dead time function" (which considers the impact of dead time), and a spectral noise function. In the time delay space, the ACF noise function, the real velocity ACF, and the dead time ACF are all merged. This product is identical to the frequency space convolution. On the basis of the observed arrival times, the ACF of the measured sample function is precisely the sum of the ACFs for dead time and noise.

In order to streamline the recorded power spectrum, the observed velocity ACF is divided by the measured sampling function ACF. Implemented within time delay space. It is possible to obtain the corrected measured power spectrum by Fourier transforming the adjusted calculated F. It is believed that the combination of the dead time function and the noise function can be regarded as components of a more comprehensive sampling function, which encompasses all factors that influence the sample rate and introduce bias into the power spectrum being measured. Altering the sampling rate may also be accomplished through instrument-specific operations, such as quantization or compression of the data. We employed two distinct spectrum estimators in our experiments: the straight technique and the slotted autocovariance method. By employing a novel approach, we successfully obtained the slotted autocovariance with significantly reduced computation time and equivalent spectrum quality to that of prior algorithms. Deconvolution appears to perform admirably with realistic computer-generated data. Dead time has little impact in both scenarios, and the practical dynamic range expands substantially until noise dominates the spectrum.

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